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February 15, 1986 to February 14, 1987

STUDIES IN THE COMPUTATION OF COMPRESSIBLE VISCOUS FLOWS

Submitted to:

Air Force Office of Scientific Research

Submitted by:

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## Interim Scientific Report

### STUDIES IN THE COMPUTATION OF COMPRESSIBLE AND VISCOUS FLOWS

AFOSR Grant No. 83-0071

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#### SUMMARY

Report here are the research activities supported by AFOSR Grant No. 83-0071 from February 15, 1986 to February 14, 1987.

On VISCOUS EFFECT INJECTION: a code has been assembled for either globle or local computation of flow over an airfoil. Using this code, we were able to show that viscous effects computed on a seperate, local grid can be injected into an inviscid solution of the Euler equations on a global grid. A paper on this work was presented at the AIAA 25th Aerospace Sciences Meeting in Reno last January. On REFINED TRANSONIC FLOW COMPUTATIONS: our computations of the flow over a circular cylinder with local refinement and an local analysis by Dr. Sobieczky, who visited and colaborated with us last summer, indicated the existence of shock free flow over a range of Mach number slightly greater than the critical value. Also our computations of the flow over an ellipse at an angle of attack showed that the circulation may not be arbitrary and is very sensitive to the amount of artificial viscosity and the grid. On THREE-DIMENSIONAL UNSTEADY TRANSONIC FLOW COMPUTATION WITH TEI: the modification of computer codes for this study has been completed. Preliminary results showed that a steady fine grid solution can be recovered on a coarse grid and the truncation error injected to render the fine grid solution is indeed very small except near the leading edge, the trailing edge, and the shock.

Three papers on our earlier works have been accepted for journal publication. Five masters/Ph.D. students have been supported fully or partially by this grant. The paper presented at the AIAA meeting is appended, and a brief description of the progress on each topic follows.

#### VISCOUS AND INVISCID INTERACTIONS

trary curvillinear meshes.

implement the method of viscous effect and truncation error injection for computations of compressible flow over an airfoil. Viscosity can be turned on and off by a parameter in the code. Communications between solutions on different grids, i.e., the extraction of the boundary conditions for the local solution and the computation of truncation error for the refined global solution, are handled by an interpolation scheme designed for arbi-

The thin-layer Navier-Stokes/Euler code, due to Steger and Pulliam, has been modified to

This code and our viscous-inviscid interaction method were applied to compute viscous flow over a NACA0012 airfoil at subcritical Mach numbers. The results showed that viscous effects, as well as truncation error, can be injected into an inviscid solution of the Euler equations on a grid which normally is too coarse to account for the viscous effects in the boundary layer. However, we have not been able to move the boundaries of a local grid arbitrarily close to the body, due to an instability of the local solution. It is found in our studies that it is better, and more general, to prescribe pertubations of the Riemann invariants as boundary conditions than the Riemann invariants. More work needs to be done to overcome the difficult of imposing proper boundary conditions for the local solution.

A summary of this work was presented at the AIAA 25th Aerospace Sciences Meeting, January, 1987 in Reno, Nevada. A copy of the paper is in Appendix I.

#### REFINED TRANSONIC FLOW COMPUTATIONS

Our preliminary results of the flow over a cylinder at Mach numbers slightly greater than the critical value M = 0.4 showed that there seems to be a range of Mach number in which the flow is free of shocks, as Van Dyke conjectured by showing the convergence of a Rayleigh-Janzen type expansion carried to very high order. By a study of the possible local, near sonic solutions

in the hodograph plane, Sobieczky has found a singular solution which may bridge the gap between smooth shock-free flow and flow with embedded shocks. Though singular, this solution does not have the post-shock logarithmic singularity described by Oswatitsch and Zierep. Further refined computations of the local flow should reveal the existence of this singularity, which could have significant effects on the stability of the boundary layer.

We have carried out a series of computations of flow over ellipses at an incident with the free stream. Since the ellipse does not have a sharp trailing edge, the flow over it may assume any circulation, according to inviscid theory. However, the circulation of such a flow computed by a numerical scheme may depend sensitively on the truncation error due to discretization and the numerical viscosity added in order to stabilized the scheme. We have found converged numerical solutions with a negative lift for an ellipse at a positive angle of attack with the free stream. We have found that the circulation depends on the grid and the amount of numerical viscosity but surprisingly, not so much on the discretization method. It is hope that further studies and more refined computations will lead to a better understanding of the numerical solution of the Euler equations.

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A Truncation Error Injection Approach to Viscous-Inviscid Interaction

B.D. Goble and K.-Y. Fung The University of Arizona, Tucson, AZ

# AIAA 25th Aerospace Sciences Meeting

January 12-15, 1987/Reno, Nevada

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### A TRUNCATION ERROR INJECTION APPROACH TO VISCOUS-INVISCID INTERACTION

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#### Abstract

A numerical procedure which uses truncation error injection derived from a local fine grid solution or from a near body viscous solution is presented in the context of solving flow over an airfoil. A global solution on a coarse grid interacts with a solution calculated on a local fine grid which is fully enclosed within the global grid. The global solution passes boundary conditions and initial values to the local solution which, in turn, is used to form truncation error approximations. Truncation error is formed by operating with the global solution operator on the local grid solution which has been interpolated onto the coarse grid. The resulting space varying function is used as a forcing function in the explicit part of the global solver. This space varying function will be nonsero only in the region common to the local grid and the global grid. Both the local and the global solutions can be viscous or inviscid. The validity of this method is confirmed by comparison with results from a thin-layer Navier-Stokes code on a NACA 0012 airfoil at a low angle of attack.

#### I. Introduction

Computation of 2-D transonic flows over aerodynamic bodies has reached a stage where solutions to most reasonable problems can be found with some degree of accuracy. Naturally, there are many available approaches to finding these solutions. One of the more popular methods is to solve the Reynolds-averaged Navier-Stokes equations using finite difference methods. An advantage of this approach is that it can compute separated flows which have vorticity convecting into the inviscid region. Also, the code tends to be very robust, able to calculate a wide variety of cases without having to change more than a few parameters. However, due to the wide range of length scales present in the problem and the difficulty of producing an adequate grid to resolve them, these codes are very expensive when Section IV contains some details on the current implemenit comes to computer resources and require a fast computer tation of the approach and Section V discusses the boundwith extensive memory in order to be practical.

flow problems is often called viscous-inviscid interaction. A compressible boundary layer code is coupled with some type of inviscid flow solver through a complex interaction

is not subject to copyright protection in the United States. symbolic approach will be used. Suppose that the differen-

scheme. The boundary layer code calculates the viscous flow near the body and passes boundary conditions to the inviscid flow solver, usually a full potential code, which in turn calculates the outer flow and passes boundary conditions back to the boundary layer scheme. The boundary conditions which are typically passed are displacement thickness or transpiration velocity from the boundary layer code to the potential code and wall pressure distribution from the potential code back to the boundary layer code. This approach is much more efficient than the Navier-Stokes solvers, both in CPU time and in memory requirements. Also, since two grids are used, it is easier to tailor the grids to capture the relevant length scales in each region. However, this approach is not as robust as the Navier-Stokes approach. The algorithm breaks down in cases with large separation regions since the boundary layer code cannot calculate this flow situation. Also, the potential flow code cannot convect vorticity away from the body or wake region and tends to overpredict shock strengths. To counteract these last two problems, Steger and Van Dalsem<sup>1</sup> incorporated a vector potential code due to Chaderjian<sup>2</sup> in place of the usual full potential code, thus including vorticity transport in the free field and giving more accurate shock fitting. New boundary conditions were also used for the inviscid part of the interaction. The boundary layer equations generate vorticity, entropy, and changes of stagnation enthalpy effects which are used as inputs to the inviscid flow solver.

This paper discusses an approach to viscous-inviscid interaction which is similar to that used by Steger and Van Dalsem. This approach is based on truncation error injection, which will be explained first. Then its application to viscous-inviscid interaction will be discussed followed by a brief discussion of the 2-D interpolation scheme used to restrict the fine grid solutions to the global coarse grid. ary conditions being used. Section VI presents the inviscid Another approach to solving these types of transonic results from a NACA0012 airfoil test case and then the viscous results.

#### II. Theory

Truncation error occurs when a differential equation is discretised on a grid with finite mesh spacing. To illus-This paper is declared a work of the U.S. Government and trate what is meant by truncation error injection, a simple

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$$L\Phi = 0 \tag{2.1}$$

where L is a continuous differential operator and  $\Phi$  is the exact continuous solution. Now finite difference L on some grid with characteristic mesh spacing h to get

$$L_h \Phi - T.E._h = 0 \tag{2.2}$$

where  $L_h$  is a discrete operator and  $T.E._h$  is the truncation error associated with the differencing. Typically, in solving this equation numerically,  $T.E._h$  is assumed to be small and is neglected. However, it can be significant. If  $T.E._h$ was known a priori, the exact solution could be found. When truncation error injection is used,  $T.E._h$  is estimated using information from a finer grid.

First, difference L on a grid with characteristic mesh spacing k, where k < h, to get

$$L_h \Phi - T.E._h = 0. \tag{2.3}$$

Now neglect  $T.E._h$ , which is smaller than  $T.E._h$ , and solve to get a solution  $\Phi_h$  that approximates  $\Phi$  to  $O(k^n)$  where n is the order of the scheme. Now restrict  $\Phi_k$  onto the coarse grid to get  $\Phi_k^h$ . Operate on the resulting solution vector with the coarse grid operator to define a new quantity T.E.? as

$$T.E.^h_h = L_h \Phi^h_h \tag{2.4}$$

where  $T.E._{h}^{h}$  is a spatially varying function which is an estimate of the local truncation error. Now inject the estimated truncation error onto the right hand side of the original equation as a forcing function and solve

$$L_h \Phi_h = T.E._h^h. \tag{2.5}$$

Obviously, if the fine grid and coarse grid are the same,  $\Phi_h$  will converge to  $\Phi_h^h$  and nothing is gained. However, suppose the fine grid is strictly a local grid, say around a shock or encompassing a boundary layer or some other region of large gradients. The function  $T.E._{h}^{h}$  will then be nonsero only at the coarse grid nodes lying within the boundaries of the local fine grid. In this case,  $\Phi_h$  will converge to a solution influenced by  $\Phi_k^h$  but not identical dependent upon such factors as the size of the patch and boundary conditions used. However, the overall solution will be improved by the injected truncation error in the patch since  $\Phi_h^h$  is a better solution than  $\Phi_h$  in the patch.

teraction scheme. Suppose that the fine grid solution is a method due to its familiarity, ease of use and the fact that viscous solution, provided by either a boundary layer solver it performs both inviscid and viscous simulations. Details or a Navier-Stokes solver, and the global solver is an in- of this code will not be presented here as it is discussed at viscid solver such as an Euler solver or a vector-potential length elsewhere.8-6 The code was modified to iteratively

tial equation to be solved can be written in operator form solver. The truncation error term formed in Eqn. (2.4) will then contain viscous information as well as truncation error information. The injection of this term as a forcing function will cause the flow to feel the viscous effects as calculated on the fine grid. The usage of two uncoupled grids also enables the relevant length scales to be captured more efficiently than if only one grid were used.

> The term  $T.E._k^h$  also contains information about the truncation error incurred in differencing the inviscid terms. This information will improve the accuracy of the convective terms on the coarse grid. Accuracy is lost in the boundary layer when the grid is coarsened in the normal direction. The truncation error information helps to reduce the loss of accuracy due to the increased mesh spacing.

#### III. Interpolation Scheme

In the implementation of this scheme, the fine grid solution must be interpolated from the local grid in one curvilinear coordinate system to the global grid in another curvilinear coordinate system and back again. This interpolation is handled by a routine that will interpolate a function of two variables from one general grid to another general grid. Some function Q1 is known as Q1(X1, Y1)and as  $Q1(\xi, \eta)$ , where (X1, Y1) are the physical coordinates of grid 1 and  $(\xi, \eta)$  are the coordinates in the corresponding computational plane. First of all, each point in grid 2 is located in grid 1 with a nearest neighbor search routine. The mappings  $Q1(\xi,\eta)$ ,  $X1(\xi,\eta)$ , and  $Y1(\xi,\eta)$ are approximately found using a local bicubic surface to represent the evenly spaced data. The point in grid 1 closest to the current point in grid 2 is used as the origin of the bicubic surface and as an initial guess to a Newton type sero finding routine. Say  $(\hat{\xi}, \hat{\eta})$  are the coordinates of the nearest neighbor point. Then  $X1(\hat{\xi}, \hat{\eta})$  and  $Y1(\hat{\xi}, \hat{\eta})$  are found as well as the partial derivatives with respect to  $\xi$ and n at that point. This new data is used in the Newton routine to drive to sero the functions

$$f(\hat{\xi},\hat{\eta}) = XPT - X1(\hat{\xi},\hat{\eta})$$

$$g(\hat{\xi},\hat{\eta}) = YPT - Y1(\hat{\xi},\hat{\eta})$$

where (XPT, YPT) are the coordinates of the current point in grid 2. At convergence, we know the coordinates  $(\xi^{\bullet}, \eta^{\bullet})$  of the current grid 2 point in the computo it. The difference between  $\Phi_h$  and  $\Phi_h^h$  in the patch is tational domain. Using the mapping found earlier, the value  $Q2(\xi^*, \eta^*)$  can now be found.

#### IV. Implementation

The thin-layer Navier-Stokes/Euler code ARC2D, due Now, look at how this approach can apply to an in- to Steger and Pulliam, is being used to implement the

#### V. Boundary Conditions

The boundary conditions being used in the code are very similar to those used in the original code.<sup>6</sup> The wall and wake conditons are the same. The far field characteristiclike boundary conditions are slightly modified so that they may also be used for the local grid. As used in Ref. 6, the locally one dimensional Riemann invariants are

$$R_1 = V_n - \frac{2a}{\gamma - 1}$$

$$R_2 = V_n + \frac{2a}{\gamma - 1}$$

where  $V_n$  is the velocity normal to the far field boundary. For an inflow boundary,  $V_n < 0$  and  $R_1$ , the tangential velocity,  $V_t$ , and an approximation to the entropy,  $S = \frac{P}{e^2}$ are all specified from free stream conditions. R2 is extrapolated from the interior. For outflow,  $V_n > 0$  and  $R_2$ ,  $V_t$ and S are extrapolated from the interior.  $R_1$  is then specified from freestream conditions. From these quantities, the four conservative variables  $\rho$ ,  $\rho u$ ,  $\rho v$ , and e can be found.

This approach has been modified slightly. Define perturbation Riemann invariants as

$$\Delta R_1 = \Delta V_n - \frac{2\Delta a}{\gamma - 1}$$
$$\Delta R_2 = \Delta V_n + \frac{2\Delta a}{\gamma - 1}$$

where, for example,

$$\Delta V_n = V_n^N - V_n^{N-1}$$

conditions for  $\Delta R_1$  and at the first grid line inside the were then compared with the fine grid viscous solution

$$V_t^N = V_t^{N-1}$$
$$S^N = S^{N-1}$$

evaluated at the boundary and for outflow we have

$$V_t^N = V_t^{N-1} + \Delta V_t$$
$$S^N = S^{N-1} + \Delta S$$

evaluated as previously described. These boundary conditions give virtually identical results as the original versions when used for far field conditions in a single grid calculation and give better results when used for the outer boundaries of the local grid.

#### VI. Results

The method was applied to a NACA0012 airfoil at  $M_{\infty} = 0.6$ ,  $\alpha = 1.0 \deg$  on a coarse grid (193x33 points) and a fine grid ( 225x49 points ) which cover the same physical domain. Viscous and inviscid cases were run on each grid for comparison with the results from the present investigation. The cases on the fine grid were interpolated to the coarse grid using the interpolation routine. These solutions were then used to form the forcing terms for the coarse grid calculations. Three calculations were made, an inviscid run forced by the fine grid inviscid solution (FIICGR3), an inviscid run forced by the fine grid viscous solution (FVICGRS), and a viscous run forced by the fine grid viscous solution (FVVCGR3). An inviscid run was also made on the fine grid using the fine grid viscous solution to form the forcing function (FVICGR1).

Coefficients of Lift, Drag, and Moment				
	$C_L$	$C_D$	CM	
ICGR1	0.15827	0.00015	-0.00081	
FIICGR3	0.15816	0.00010	-0.00071	
VCGR1	0.14555	0.00197	0.00095	
FVICGR3	0.14558	0.00194	0.00093	
FVVCGR3	0.14558	0.00194	0.00093	
FVICGR1	0.14555	0.00197	0.00095	

TABLE 1

 $M_{\infty} = 0.6$ ,  $\alpha = 1.0 \deg$ , inviscid. Global and Local grids have same domain.

These four runs were made using the fine grid solutions as initial conditions to check the interpolation routine and the implementation of the method. Each calculation conand  $V_n^N$  is evaluated at the  $N^{th}$  time step at freestream verged in one step as it should. These resulting solutions

all the cases. The agreement between the appropriate cases is quite good. is good. Therefore, we can conclude that the interpolation routine is working well and we can proceed.

The next step was to implement local to global interaction to solve inviscid flow about a NACA0012 airfoil at  $M_{\infty} = 0.5$ ,  $\alpha = 2.0 \deg$ . A 161x33 grid with a minimum  $\Delta Y = 0.002$  was used for the global grid. Several different local grids were used. They were all derived from either a 225x49 grid with a minimum  $\Delta Y = 0.0005$  or a 249x65 grid with a minimum  $\Delta Y = 0.0001$ . The local grids were formed by taking the first n lines from the body in the full grid. Each of the cases was restarted using the inviscid solution on the global grid as an initial condition. The following tables show values for  $C_L$ ,  $C_D$  and  $C_M$  as found on the local grid.

Coefficients of Lift, Drag, and Moment					
	$C_L$	$C_D$	C <sub>M</sub>		
225x49	0.27847	0.00034	-0.00176		
225x37	0.27824	0.00035	-0.00176		
225x35	0.27835	0.00034	-0.00176		
225x33	0.27847	0.00035	-0.00177		
225x31	0.27845	0.00034	-0.00177		
225x29	0.27851	0.00038	-0.00179		
225x27	0.27845	0.00040	-0.00180		

TABLE 2

 $M_{\infty} = 0.5$ ,  $\alpha = 2.0 \deg$ , Inviscid. Local grid based on 225x49 grid. Min  $\Delta Y = 0.0005$ 

Table 2 shows the lift and drag coefficients for the cases using local grids based on the 225x49 grid. The first entry in the table is the single grid solution obtained on the full 225x49 grid. The next two cases agree well with these results. Beginning with the third case, the values start to diverge from the single grid results. The values for  $C_L$  do not appear to follow any logical progression as the size of the fine grid is reduced, but all values shown are within 0.1% of the 225x49 case. Actually, the closest solution is the 225x33 case. No explanation for this behavior has been found. A 225x25 case was also attempted. The solution appeared to converge very well and then suddenly diverged very rapidly. It was determined that this behavior was due to the fine grid outer boundary being too close to the airfoil.

Fig. 1 shows the Mach number contours for the 225x31 case as displayed on the global grid. The gridline shown is the approximate outer limit of the local grid. The contours are continuous across the grid boundary indicating

( VCGR1 ) and the fine grid inviscid solution ( ICGR1 ) solution. Comparing Fig. 1 with Fig. 2, which shows the in terms of  $C_L$ ,  $C_D$ , and  $C_M$ . Table 1 shows the  $C_L$ ,  $C_D$  Mach number contours as calculated on the 249x65 grid, and  $C_M$  values due to integrating  $C_P$  around the body for we can see that the agreement with the fine grid solution

Coefficients of Lift, Drag, and Moment				
	CL	$C_{\mathcal{D}}$	C <sub>M</sub>	
249x65	0.27892	0.00030	-0.00175	
249x47	0.27857	0.00030	-0.00175	
249x45	0.27876	0.00031	-0.00177	
249x43	0.27847	0.00035	-0.00177	
249x41	0.27845	0.00034	-0.00177	
249x39	0.27851	0.00038	-0.00179	

TABLE 3

 $M_{\infty} = 0.5$ ,  $\alpha = 2.0 \deg$ , Inviscid. Local grid based on 249x65 grid. Min  $\Delta Y = 0.0001$ 

Table 3 shows the lift and drag coefficients for the cases using local grids based on the 249x65 grid. The first entry is the single grid result for the 249x65 grid. The results shown exhibit the same pattern as the previous case. The best agreement in  $C_L$  occurred on the 249x45 grid. All  $C_L$  values are within 0.2% of the fine grid value. The accuracy for the  $C_D$  and  $C_M$  values does increase as the sise of the fine grid is increased. In this case, the smallest grid that could be used without the solution diverging was a 249x39 grid. Fig. 3 shows the  $C_P$  contours for the 225x45 case. Again, the solution appears to be continuous across the local grid boundary. The CP contours for the 249x65 case are shown in Fig. 4. Agreement between the two cases is good. Fig. 5 shows the entropy contours for the 249x45 case. All entropy generation is taking place at the leading edge or elsewhere along the body. The local grid boundary is not generating any entropy. Fig. 6 shows the entropy contours for the 249x65 case.

Coefficients of Lift, Drag, and Moment				
	$C_L$	$C_D$	C <sub>M</sub>	
225x49	0.25084	0.01098	0.00232	
225x39	0.25141	0.01095	0.00234	
225x37	0.25133	0.01094	0.00234	
225x35	0.25135	0.01092	0.00234	
225x33	0.25081	0.01094	0.00232	
225x31	0.25118	0.01098	0.00228	
193x49	0.24974	0.01261	0.00256	

TABLE 4

 $M_{\infty} = 0.5$ ,  $\alpha = 2.0 \deg$ ,  $Re = 10^6$ . Local grid based on 225x49 grid. Min  $\Delta Y = 0.0005$ 

The code was next applied to subcritical viscous flow that the local solution is blending smoothly with the global about a NACA0012 airfoil at  $M_{\infty}=0.5$ ,  $\alpha=2.0$  deg, and Re = 106. A 193x49 global grid with a minimum putational Fluid Mechanics Laboratory of the Aerospace tom of the airfoil.

The values for  $C_L$ ,  $C_D$  and  $C_M$  for the viscous cases are shown in Table 4. Again the values of  $C_L$  do not follow any recognisable pattern as the local grid size is changed. The best agreement occurs in the 225x33 case; all others are within 0.3% of the single grid solution. Unlike the inviscid results, here the  $C_D$  and  $C_M$  values also do not follow any sort of pattern. The 225x33 case gives the best overall agreement for the three coefficients. The values for the 193x49 case shown are for the solution as found on the global grid alone without a local grid and were included for comparison. Fig. 7 shows the Mach contours from the 225x33 case. Note the smoothness of the contours across the local grid boundary and the good agreement with the single grid results shown in Fig. 8. The plots of C<sub>P</sub> shown in Fig. 9 and Fig. 10 also agree very well.

#### VII. Conclusions

These results show the validity of this approach to solving inviscid or viscous flow over airfoils. More work needs to be done to understand better where to place the outer boundary of the local grid. The location of the outer boundary of the local grid, beyond some threshold, does not appear to have any predictable effect on the accuracy of  $C_{I}$ . Except in the viscous case, increasing the size of the local grid did improve the results for  $C_D$  and  $C_M$ . Also, with more understanding on how the two grids interact, better grids can be generated to put more resolution where it is needed.

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search Center, Moffett Field, California and at the Com- AIAA Paper No. 78-257(1978).

 $\Delta Y = 0.00006$  was used in conjunction with local grids deared Mechanical Engineering Department at the Univerrived from a 225x49 grid with a minimum  $\Delta Y = 0.00002$ . sity of Arisona. The authors would like to thank NASA The global solution was found using the Euler solver and for its help and support in conducting the work presented viscous type boundary conditions while the local solution here. We are grateful to NASA for support under Interwas found assuming turbulent viscous flow. An algebraic change Number NCA2-107, and to the Air Force under mixing length model due to Baldwin and Lomax was used Contract Number AFOSR-83-0071. We are also grateful to to approximate the turbulent effects. Transition was as- Dr. Terry Holst for his permission to use the NASA Ames sumed to occur at 10% of chord on both the top and bot- CRAY-XMP computer for this and other studies related to the traineeship program. We are especially grateful to Dr. Tom Pulliam for his guidance and instruction during the first author's time at Ames.

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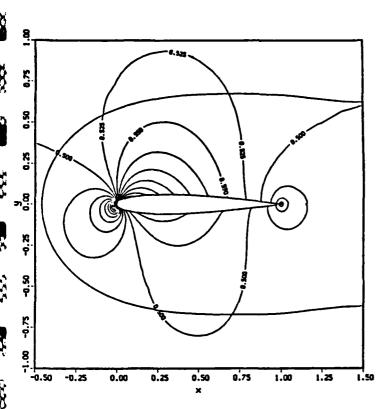


Fig. 1 Mach Number Contours.  $M_{\infty} = 0.5$ ,  $\alpha = 2$  deg, 225x31 local grid, 161x33 global grid.

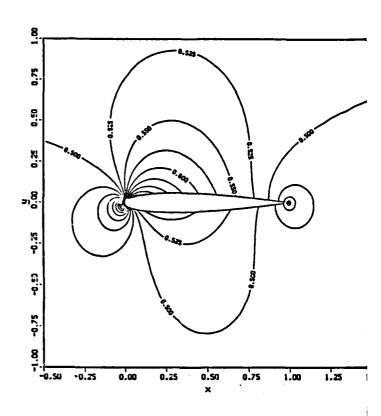


Fig. 2 Mach Number Contours.  $M_{\infty} = 0.5$ ,  $\alpha = 2 \text{ deg}$ , 249x65 single grid.

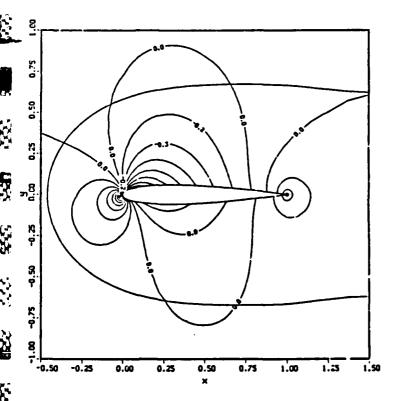


Fig. 3  $C_p$  Contours.  $M_{\infty} = 0.5$ ,  $\alpha = 2$  deg, 249x45 local grid, 161x33 global grid.

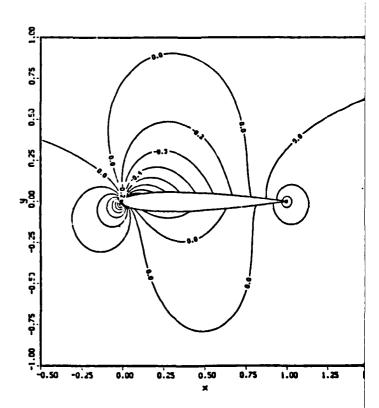


Fig. 4  $C_p$  Contours.  $M_{\infty} = 0.5$ ,  $\alpha = 2$  deg 249x65 single grid.

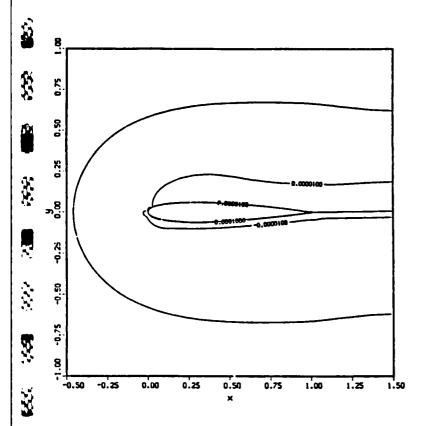


Fig. 5 Entropy Contours.  $M_{\infty} = 0.5$ ,  $\alpha = 2 \text{ deg}$ , 249x45 local grid, 161x33 global grid.

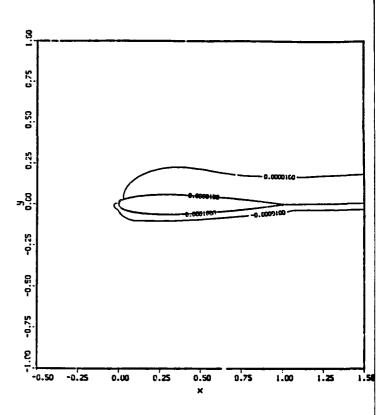


Fig. 6 Entropy Contours.  $M_{\infty} = 0.5$ ,  $\alpha = 2 \text{ deg}$ , 249x65 single grid.

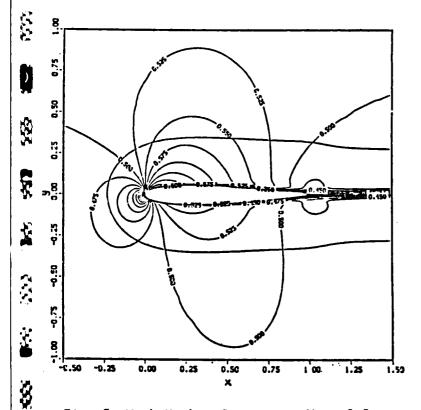


Fig. 7 Mach Number Contours.  $M_{\infty} = 0.5$ ,  $\alpha = 2$  deg, 225x33 local grid, 193x49 global grid.

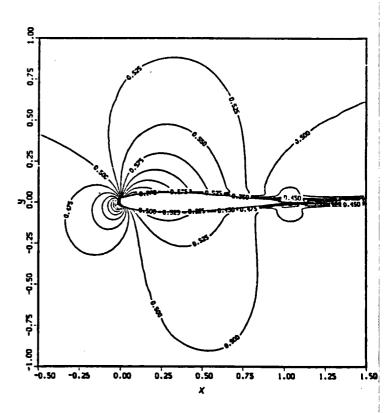


Fig. 8 Mach Number Contours.  $M_{\infty}$  = 0.5,  $\alpha$  = 2 deg, 225x49 single grid.

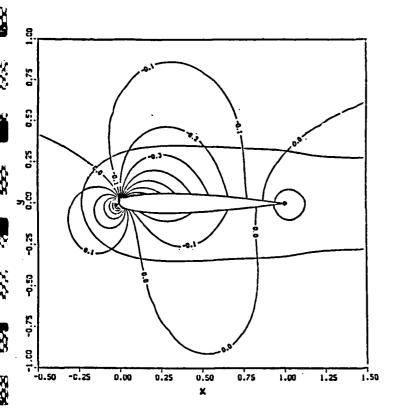


Fig. 9 Cp Contours.  $M_{\infty} = 0.5$ ,  $\alpha = 2$  deg, Re =  $10^6$ ,  $225 \times 33$  local grid,  $193 \times 49$  global grid.

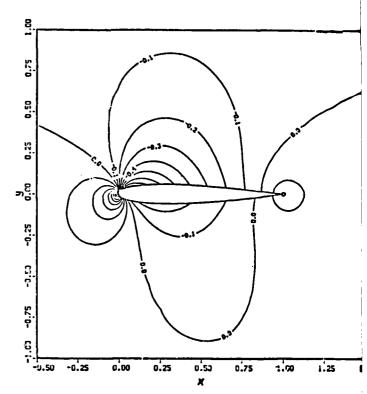


Fig. 10 Cp Contours.  $M_{\infty} = 0.5$ ,  $\alpha = 2$  deg Re =  $10^6$ , 225x49 single grid.